

Homework #1: The Solow Growth Model
ECON 5163
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1. Prove that the steady state growth rate of output in the Solow growth model is $g + n + ng$ where g is the growth rate of labor-augmenting technology and n is the population growth rate. Assume the production function is constant returns to scale.
2. How does the depreciation rate affect the balanced growth level of capital? That is, mathematically show what happens to the balanced growth level of capital when the depreciation rate rises? Repeat this analysis for an increase in both the growth rates of labor-augmenting technological progress, g , and the population growth rate, n .
3. Suppose that the production function is Cobb-Douglas, $Y_t = (K_t)^\alpha (A_t L_t)^{(1-\alpha)}$.
 - A. Express the production function, $F[K_t, A_t L_t]$, in terms of output per unit of effective labor. That is, find $f(k_t)$?
 - B. Find expressions for k^* , y^* and c^* as functions of the parameters of the model, s , n , δ , g , and α .
 - C. What is the golden-rule value of k^* ?
 - D. What savings rate is needed to yield the golden-rule value of k^* ?
4. Assume labor and capital are paid their marginal products in perfectly competitive markets. That is, $F_K[K_t, A_t L_t] = q_t$ and $F_L[K_t, A_t L_t] = w_t$.
 - A. Show that the marginal product of labor, $F_L[K_t, A_t L_t]$, is equal to $A_t[f(k_t) - k_t f'(k_t)]$.
 - B. Show that under a constant returns to scale production function, $w_t L_t + q_t K_t = F(K_t, A_t L_t)$.
 - C. The rental rate of capital, q_t , is roughly constant over time, while the real wage, w_t , tends to rise. Does a Solow economy on a balanced growth exhibit these paths? That is, calculate the growth rates of w_t and q_t on a balanced growth path?
 - D. Suppose the economy begins with a level of k_t less than k^* . As k moves toward k^* , is w_t growing at a greater than, less than, or equal to its growth rate on the balanced growth path? What about q_t ?