

Homework #2: The Ramsey Growth Model
ECON 5163
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1. Consider an individual who lives for two periods and whose utility is given by
$$U(C_1, C_2) = [C_1^{(1-\theta)}]/(1-\theta) + \beta[C_2^{(1-\theta)}]/(1-\theta).$$
Let P_1 and P_2 denote the prices of consumption in the two periods and let W denote the value of an individual's lifetime income. Thus, the budget constraint is $P_1C_1 + P_2C_2 = W$.
 - A. What is the individual's utility maximizing choices of C_1 and C_2 given P_1 , P_2 , and W ?
 - B. The elasticity of substitution between consumption in the two periods is $-\left[\frac{P_1/P_2}{C_1/C_2}\right]\left[\frac{\partial(C_1/C_2)}{\partial(P_1/P_2)}\right]$. Show that with the utility function above, the elasticity of substitution between C_1 and C_2 is $1/\theta$.
2. Consider the production function $Y_t = (K_t)^\alpha(A_tL_t)^{(1-\alpha)}$ or $f(k_t) = (k_t)^\alpha$. Assume that firms hire labor for w_tA_t and rent capital for q_t . Show that the cost minimizing level of k_t depends only on w_t , q_t , and α .
3. Consider a Ramsey growth model that is on its balanced growth path where the government has policy of taxing income derived from capital rents instead of imposing a lump-sum tax. If the tax rate is τ , the real rental rate of capital, q_t , that households earn from capital is $q_t = (1 - \tau)f'(k_t)$. We will assume the government returns this revenue in the form of lump-sum transfers, t_t , to the households. That is, $\tau f'(k_t)k_t = t_t$.
 - A. How does this tax affect the $c_{t+1} - c_t$ curve? How about the $k_{t+1} - k_t$ curve?
 - B. How do the values of c and k on the new balanced growth path compare with their values on the balanced growth path for a model without a government?
 - C. How do your answers to parts (A) and (B) change if the government does not rebate the revenue from the tax but instead uses it to make government purchases? That is, $\tau f'(k_t)k_t = g_t$?