Homework #4: New Growth Theory
ECON 5163
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1. Consider the research and development model of growth with labor, capital, and technology. Assume that $\beta + \theta < 1$ and $n > 0$, and that the economy is on the balanced growth path. Describe how each of the following changes affects the $dG^A = 0$ and $dG^K = 0$ lines and the position of the economy in the (G^A, G^K) space at the moment of the change:

   A. An increase in n.

   B. An increase in a_K.

   C. An increase in θ. (Hint: Take the natural log of the G_t^K and G_t^A equations and then find $\partial \ln(G_t^K)/\partial\theta$ and $\partial \ln(G_t^A)/\partial\theta$)

2. Consider the research and development model of growth with labor, capital, and technology. Assume that $\beta + \theta = 1$ and $n = 0$.

   A. Using the capital and technology growth rate equations, find the value that A/K must have for G^K and G^A to be equal.

   B. Using your result in part (A), find the growth rate for A and K when $G^K = G^A = G^*$.

   C. How does an increase in *s* affect the long-run growth rate of the economy?

   D. What value of a_K maximizes the long-run growth rate in the economy? (Hint: Take the natural log of the G* equation and then find $\partial \ln(G^*)/\partial a_K$.)

3. Suppose that output and technology are determined by the learning-by-doing model present in class. Suppose $\varphi < 1$. Given that $G^K = n/(1 - \varphi)$, what are the balanced growth path values of G^Y and G^A?

4. Suppose that output is given by $Y_t = [K_t]^\alpha [A_t L_t]^{(1-\alpha)}$ where L_t is constant and equal to 1. Furthermore, suppose capital accumulation is given by $K_{t+1} - K_t = sY_t$, and technology accumulation is given by $A_{t+1} - A_t = BY_t$ where B is a constant.

   A. Find expressions for G_t^A and G_t^K in terms of A_t, K_t, and the parameters.

   B. Sketch the $dG^A = 0$ and $dG^K = 0$ lines in the (G^A, G^K) space.

   C. Does the economy converge to a balanced growth path? If so, what are the growth rates of K_t, A_t, and Y_t on the balanced growth path?

   D. How does an increase in *s* affect long-run growth?