Homework #4: Computationally Solving Rational Expectations Models
ECON 6313
Benjamin Keen

(**Include copies of all your Matlab code**) Let us consider the neoclassical growth model from Homework #3. At any date t, households in the economy gain utility from consumption, C_t, and disutility from labor, N_t. Those preferences are represented by the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t E_0 \left[\ln(C_t) + \theta \ln(1 - N_t) \right],$$

where β is a discount factor. Households' demands are restricted by their budget constraint:

$$Z_t (K_t)^\alpha (N_t)^{1-\alpha} = C_t + K_{t+1} - (1-\delta) K_t,$$

where K_t is capital, α is capital's share of output, and δ is the capital depreciation rate. The parameter Z_t is an exogenous parameter that represents the level of productivity in the economy. That productivity factor is a stochastic process that follows an AR(1) representation:

$$\ln(Z_t / Z^*) = \rho_Z \ln(Z_{t-1}/Z^*) + \eta_t,$$

where Z^* is the steady state value of Z_t and $\eta_t \sim N(0, \sigma^2)$ [Hint: $\ln(Z_t/Z^*) \approx \widehat{Z}_t$]. The parameters are set to the following values: $\alpha = 0.33$, $\beta = 0.99$, $\delta = 0.025$, $\rho_Z = 0.95$, $\sigma^2 = 1$, and $Z^* = 1$. Finally, the time constraint forces the steady state value of labor, N^*, to be between 0 and 1. Since the preference parameter θ is key in determining N^*, a value for θ is selected so that $N^* = 0.2$. That is, N^* is set equal to 0.2 and θ is determined from the first-order condition for labor.

1. Using the first-order conditions for C_t, K_{t+1}, N_t, and λ_t from Question 2 part c from Homework #6, find the steady steady state values for C_t, K_t, and λ_t and the value for θ [Hint: Use the first-order condition for N_t to find θ].

2. Linearize the first-order conditions for C_t, K_{t+1}, N_t, and λ_t and place them into matrix form: $A E_t \widehat{Y}_{t+1} = B \widehat{Y}_t + C_0 \widehat{X}_t + C_1 E_t \widehat{X}_{t+1}$, where $\widehat{Y}_t$ is the percent deviation of the endogenous variables from their steady state, and $\widehat{X}_t$ is the percent deviation of the exogenous variable, Z_t, from its steady state [Hint: This is the same answer as in Question 5 from Homework #6]. Use that matrix form and the King and Watson [1998, 2002] Matlab programs to find the rational expectations solution in the state space form: $Z_t = \Pi S_t$ and $S_t = M S_{t-1} + \varepsilon_t$. In your answer, you just need to specify the vectors Z_t and S_t and the resulting matrix Π from the state space solution.

3. Suppose there is a positive 1% shock to the technology factor in period 1 (i.e., $\varepsilon_1 = 1$). Plot the impulse response functions for C_t, K_t, and N_t in periods $1 - 30$. Be sure to properly label your graphs. Why is the period 1 response of capital equal to zero?

4. Calculate the 3-step ahead forecast error variance for C_t, K_t, and N_t.

5. What are the unconditional variances for C_t, K_t, and N_t?

6. Find the correlation between K_t and N_t, corr(K_t,N_t), and the correlation between K_t and N_{t-1}, corr(K_t,N_{t-1}).