Homework #5: Lucas Supply Curve and the New Keynesian Model
ECON 6313
Benjamin Keen

1. Consider the following money in the utility function model:

$$y_t = A_t n_t^{(1-\alpha)}, \tag{1}$$

$$(1-\alpha)A_t n_t^{-\alpha} = W_t/P_t, \tag{2}$$

$$E_t[P_{t+1}y_{t+1}^{\phi}] = \beta R_t P_t y_t^{\phi}, \tag{3}$$

$$\theta_N n_t^{\gamma} = y_t^{-\phi}(W_t/P_t), \tag{4}$$

$$\theta_M (M_t/P_t)^{-\upsilon} = y_t^{-\phi}(1 - 1/R_t), \tag{5}$$

$$M_t/M_{t-1} = \mu, \tag{6}$$

where y_t is output, n_t is labor, W_t is the nominal wage, P_t is the price level, R_t is the gross nominal interest rate, M_t is nominal money balances, $\mu > 1$ is the gross steady-state growth rate of nominal money balances, $0 < \alpha < 1$, $0 < \beta < 1$, $\phi > 0$, $\gamma > 0$, $\upsilon > 0$, $\theta_N > 0$, and $\theta_M > 0$. Equations (1) – (6) describe the production function, labor demand, the New Keynesian IS Curve, labor supply, money demand, and money supply, respectively. Since nominal money balances grow in the steady-state, $M = \mu M_{-1}$, show mathematically how the value of μ impacts steady-state real money balances. [Hint: You may assume the steady-state inflation rate is constant and equal to $\pi = P/P_{-1}$.]

2. Consider the following linearized model:

$$\widehat{y}_t = (1-\alpha)\widehat{n}_t + \varepsilon_t, \tag{7}$$

$$\widehat{y}_t - \widehat{n}_t = \widehat{W}_t - \widehat{P}_t, \tag{8}$$

$$E_t[\widehat{y}_{t+1} - \widehat{y}_t] = (1/\phi)(\widehat{R}_t - (E_t[\widehat{P}_{t+1}] - \widehat{P}_t)), \tag{9}$$

$$\gamma\widehat{n}_t + \phi\widehat{y}_t = \widehat{W}_t - \widehat{P}_t, \tag{10}$$

$$\widehat{M}_t - \widehat{P}_t = \widehat{Y}_t, \tag{11}$$

$$\widehat{M}_t = \rho\widehat{M}_{t-1} + e_t, \tag{12}$$

where y_t is output, n_t is labor, W_t is the nominal wage, P_t is the price level, R_t is the gross nominal interest rate, M_t is nominal money balances, $\hat{}$ is the percent deviation from steady state, $0 < \alpha < 1$, $\phi > 0$, $\gamma > 0$, $0 < \rho < 1$, $\varepsilon_t \sim N(0, \sigma_A^2)$, and $e_t \sim N(0, \sigma_M^2)$. Equations (7) – (12) describe the production function, labor demand, the New Keynesian IS Curve, labor supply, money demand, and money supply, respectively. Wage stickiness impacts the economy due to one-period nominal wage contracts, W_t^c, partially indexed to the price level, $0 \le b \le 1$, such that

$$\widehat{W}_t^c = E_{t-1}[\widehat{w}_t^*] + E_{t-1}[\widehat{P}_t] + b(\widehat{P}_t - E_{t-1}[\widehat{P}_t]), \tag{13}$$

where w_t^* is the real wage in a flexible wage economy. Calculate the Lucas Supply Curve for this economy. [Hint: You may assume $E_{t-1}[\widehat{y}_t^*] = 0$, where y_t^* is output in a flexible wage economy.]

3. The intermediate firms produce differentiated goods in a monopolistically competitive market but encounter price frictions that interfere with optimal price adjustment. Firm f hires labor, $n_{f,t}$, at a real wage rate of w_t and rents capital, $k_{f,t}$, at a real rental rate of q_t. Those labor and capital inputs are utilized by firm f to produce its output, $y_{f,t}$, according to a Cobb-Douglas production function:

$$y_{f,t} = Z_t(k_{f,t})^{\alpha}(n_{f,t})^{1-\alpha}, \tag{14}$$

where $0 \leq \alpha \leq 1$. A continuum of differentiated inputs, $y_{f,t}$, where $f \in [0,1]$ is used by the final goods firm to produce aggregate output, y_t, such that

$$y_t = \left[\int_0^1 y_{f,t}^{(\varepsilon_p - 1)/\varepsilon_p} df\right]^{\varepsilon_p/(\varepsilon_p - 1)},$$

where $-\varepsilon_p$ is the price elasticity of demand. Cost minimization by the final goods firm generates the following demand equation for firm f's good:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon_p} y_t, \tag{15}$$

where $P_{f,t}$ is the price of $y_{f,t}$, and P_t is a nonlinear aggregate price index:

$$P_t = \left[\int_0^1 P_{f,t}^{1-\varepsilon_p} df\right]^{1/(1-\varepsilon_p)}. \tag{16}$$

Each period, a fraction, $(1 - \eta_{sp})$, of intermediate goods firms can adjust their prices, while the remaining fraction, η_{sp}, of firms increase their prices by last period's inflation rate, π_{t-1}. Using the information above, derive the linearized New Keynesian Price Phillips Curve with indexation to the lagged inflation rate.