The Meaning of Interest Rates

This lecture focuses on defining and calculating interest rates.

Measuring Interest Rates

A. Present value (PV)

1. A dollar received one year from today is less valuable than a dollar received today.
2. Present value of a cash payment (CP) received *n* years from today:

$$PV = \frac{CP}{(1+R)^n}$$

where R = interest rate.

3. Example:

How much is $100 in today's dollars worth in *n* years?
(i.e., Find CP, where R = 0.08 and PV = $100)

<u>1 year</u>

$100\times(1.08) = \$108.00$

<u>2 years</u>

$108.00\times(1.08) = 100\times(1.08)^2 = \$116.64$

<u>3 years</u>

$116.64\times(1.08) = 100\times(1.08)^3 = \$125.97$

<u>*n* years</u>

$100\times(1.08)^n$

B. Four types of credit market instruments

1. Simple loan

   a. Borrower repays the lender the principle plus interest at the maturity date.

   b. Use PV formula, where PV is the initial loan, R is the interest rate, CP is the cash payment due on maturity, and *n* is the number of years until the loan reaches maturity.

2. Fixed-payment loan (a fully amortized loan)

   a. Borrower repays the lender in equal payments (part principle and part interest) every period (usually a month) for a fixed number of years.

   b. Examples: Home mortgage loans and auto loans.

3. Coupon bond

    a. The borrower pays the lender a fixed interest payment (coupon payment) every year until the maturity date when a final amount (face value or par value) is due.

    b. Four key components of a coupon bond:

        i. The bond’s face value.

        ii. The corporation or govt. agency issuing the bond.

        iii. The bond’s maturity date.

        iv. The bond’s coupon rate (coupon rate = coupon payment/face value).

4. Discount bond (zero-coupon bond)

    a. Lender pays the borrower less than face value but is repaid the face value at maturity.

    b. This bond does not make any interest payments.

C. Yield to maturity is the interest rate that equates the present value of future cash payments from a debt instrument with today's value of that instrument (i = yield to maturity).

1. Simple loan

   a. A simple loan payable in *n* years.

$$PV = \frac{CP}{(1+i)^n}$$

$$(1+i)^n = \frac{CP}{PV}$$

$$i = \left(\frac{CP}{PV}\right)^{1/n} - 1$$

   b. Example: PV = \$200, CP = \$220.50, and n = 2.

$$i = \left(\frac{220.5}{200}\right)^{1/2} - 1 = 0.05$$

2. Fixed payment loan

   a. A fixed payment loan payable for the next *n* years.

$$\text{LV} = \frac{\text{FP}}{(1+\text{i})} + \frac{\text{FP}}{(1+\text{i})^2} + \cdots + \frac{\text{FP}}{(1+\text{i})^n}$$

LV = loan value
FP = fixed payment.

   b. Example: LV = \$1,000, FP = \$85.81, and n = 25.

$$1{,}000 = \frac{85.81}{(1+\text{i})} + \frac{85.81}{(1+\text{i})^2} + \cdots + \frac{85.81}{(1+\text{i})^n}$$

Calculate i = 0.07 with a financial calculator.

3. Coupon bond

a. A coupon bond maturing in *n* years.

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

P = price of bond
C = yearly coupon payment
F = face value.

b. Example: P = \$889.20, F = \$1,000, C = \$100, and n =10.

$$889.20 = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \cdots + \frac{100}{(1+i)^{10}} + \frac{1,000}{(1+i)^{10}}$$

Calculate i = 0.1225 with a financial calculator.

c. The relationship between the coupon rate and the coupon bond price:

TABLE 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

i. When the price of a coupon bond equals face value, the yield to maturity equals the coupon rate.

ii. The yield to maturity is negatively related to the bond price such that as the yield to maturity rises (falls) the bond price falls (rises).

iii. When the yield to maturity is greater (less) than the coupon rate, the coupon bond price is below (above) its face value.

4. Discount bond

a. A discount bond maturing in *n* years.

$$P = \frac{F}{(1+i)^n}$$

$$(1+i)^n = \frac{F}{P}$$

$$i = \left(\frac{F}{P}\right)^{1/n} - 1$$

F = face value of the discount bond
P = price of the discount bond

b. Example: F = \$1,000, P = \$900, and n = 2.

$$i = \left(\frac{1{,}000}{900}\right)^{1/2} - 1 = 0.054$$

5. <u>Perpetuity (or consol)</u> is a special case of a coupon bond where coupon payments are made every period but the bond has no maturity date.

   a. Perpetuities were first sold by the British Treasury during the Napoleonic Wars and are still traded today.

   b. A perpetuity

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots$$

$$P = C\left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots\right)$$

$$P = C\left(\frac{1}{i}\right)^*$$

$$i = \frac{C}{P}$$

*See proof at the end of the notes.

c. Example: P = $1,000 and C = $50.

$$i = \frac{50}{1{,}000} = 0.05$$

The Difference between Rate of Return and Yield to Maturity

A. Rate of return

1. The rate of return is the amount of any payment in a period (ex., coupon payment) plus any change in the asset's price as a percentage of the asset's price.

2. The rate of return on a coupon bond:

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

R = rate of return on the coupon bond
C = coupon payment
P_t = price of the coupon bond at time t
P_{t+1} = price of the coupon bond at time t+1
$\frac{C}{P_t}$ = coupon rate on the bond
$\frac{P_{t+1}-P_t}{P_t}$ = rate of capital gain on the bond

3. The rate of return will not necessarily equal the yield to maturity.

B. The impact of a changing interest rate (yield to maturity) on the rate of return

TABLE 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator, using Equation 3.

1. The rate of return equals the yield to maturity only if the holding period is the same as the time to maturity. (See the last row of Table 2.)
2. An increase in interest rates causes bond prices to fall as long as the time to maturity is longer than the holding period.
3. Whenever it takes a bond longer to reach maturity, a rise in the interest rate generates a larger decline in the bond price and a lower rate of return.
4. A rising interest rate can cause the rate of return to be negative even if its coupon rate is large.

C. Maturity and the volatility of bond returns: Interest-rate risk

1. Prices and rates of return are more volatile for long-term bonds than for short-term bonds.

2. Interest-rate risk is the risk to a bond's rate of return caused by interest rate changes.

3. Long-term bonds have a much larger interest rate risk than short-term bonds.

The Difference between Nominal and Real Interest Rates

A. The nominal interest rate

1. This interest rate is the actual rate paid by borrowers and received by savers.

2. This interest rate does not account for expected inflation.

B. Real interest rate

1. This interest rate adjusts for expected inflation and more accurately reflects the cost of borrowing.

2. Fisher equation

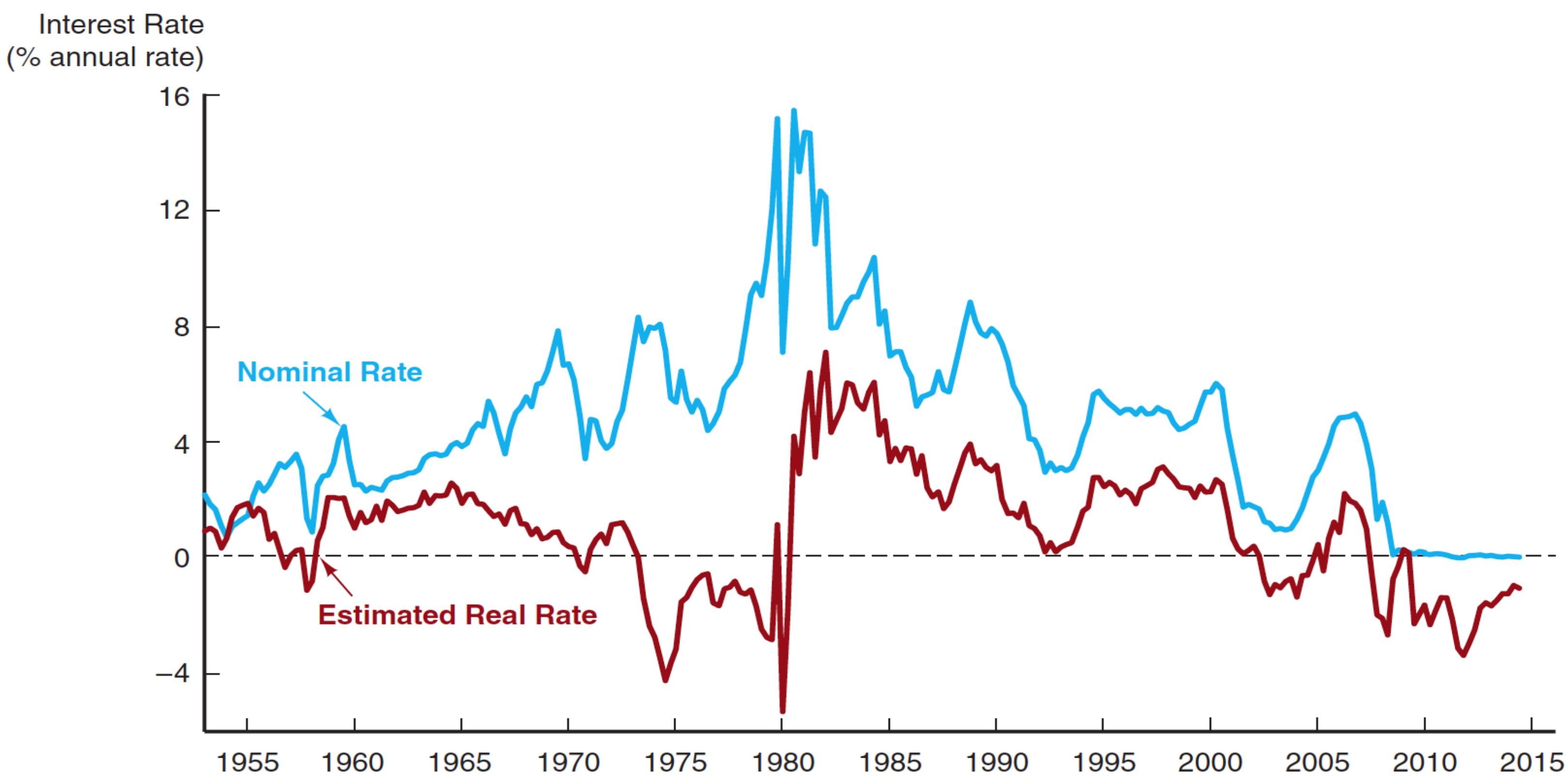
$$R = r + \pi^e$$

$$R = \text{nominal interest rate}$$
$$r = \text{real interest rate}$$
$$\pi^e = \text{expected inflation rate}$$

3. The real interest rate impacts the incentives to borrow and lend more than the nominal interest rate.

4. One problem is that precise data on the expected inflation rate does not exist.

C. The nominal and real interest rates do not always move together.



Proof:

Show $$\left(\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots\right)=\frac{1}{i}$$

$$\text{Set: } x = 1/(1+i)$$

$$\text{Formula: } 1 + x + x^2 + x^3 + \cdots = 1/(1-x)$$

$$\left(1+\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots\right)=\frac{1}{1-\frac{1}{(1+i)}}$$

$$\left(\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots\right)=\frac{1}{\frac{1+i-1}{(1+i)}}-1$$

$$\left(\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots\right)=\frac{(1+i)}{i}-\frac{i}{i}$$

$$\left(\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots\right)=\frac{1}{i}$$