The Risk and Term Structure of Interest Rates

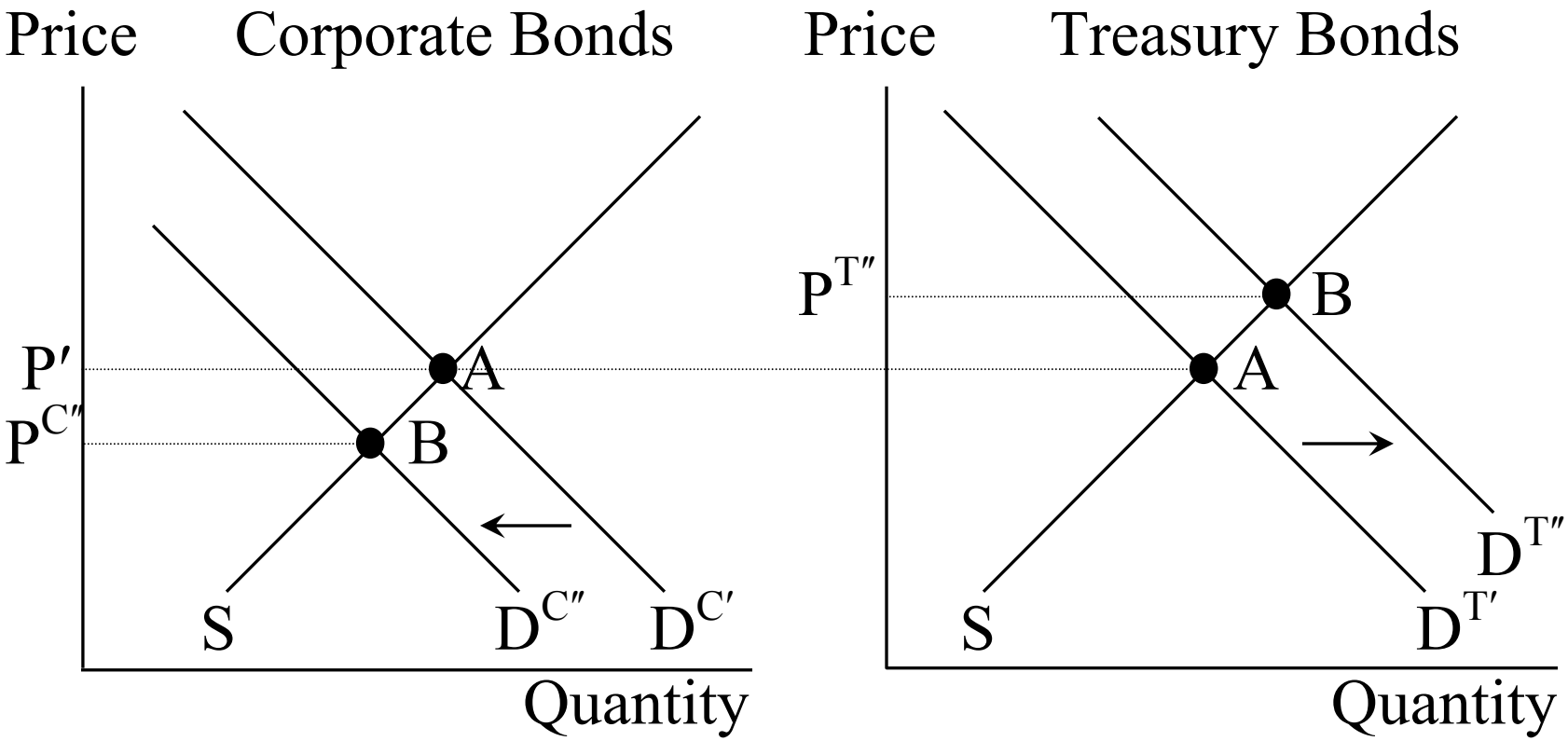
This lecture examines why bonds with the same and different terms to maturity have different interest rates.

Risk Structure of Interest Rates

A. Default risk

1. The default risk accounts for the probability the issuer of a bond is unable or unwilling to make interest payments on time or to pay the face value of the bond at maturity.
2. Default-free bonds are bonds, such as U.S. Treasuries, that have no default risk.
3. Risk premium is the spread between the interest rate on a bond with a default risk and the interest rate on a default-free bond (same maturity). Essentially, it measures the additional interest people must earn to hold the risky bond.

4. Example: Suppose a corporate bond with a zero risk premium has an increase in default risk.



a. Demand for corporate bonds (D^C) declines.

b. Demand for Treasury bonds (D^T) rises.

c. Risk premium is $P^{T''} - P^{C''}$.

5. Key points about the risk premium
    a. A bond with a default risk will always have a positive risk premium.
    b. An increase in a bond's default risk will raise the risk premium.
6. Credit-rating agencies (ex., Moody's, S&P, and Fitch) are investment advisory firms that rate the quality of bonds in terms of their default risk.
    a. AAA bonds are considered default-free bonds.
    b. Junk bonds are dubbed speculative grade bonds with a high default risk.

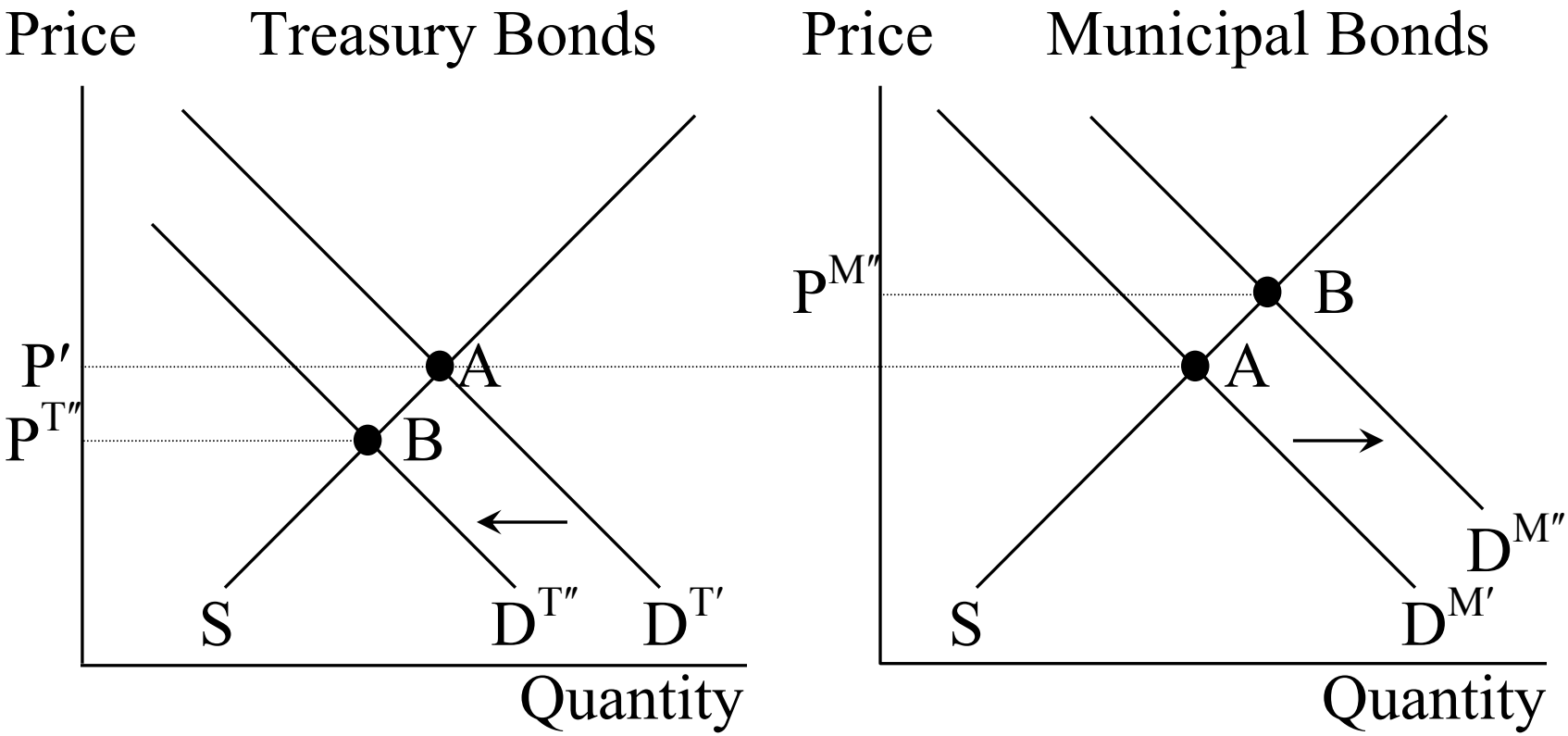
B. Liquidity

1. Liquidity is the relative ease with which a bond can be converted into cash.
2. Bonds with more liquidity are preferable to bonds with less liquidity.
3. U.S. Treasury bonds are the most liquid of all bonds because they are so widely traded.

C. Income tax considerations

1. Interest earned from municipal bonds is exempt from federal income taxes while interest from U.S. Treasury bonds is not. That raises the demand for municipal bonds which pushes down the interest rate on municipal bonds.
2. Example: The 2013 income tax increases raised the demand for municipal bonds and lowered demand for U.S. Treasury bonds.

3. Example: Suppose a municipal bond moves from being taxable to tax exempt.



a. Demand for Treasury bonds (D^T) declines.

b. Demand for municipal bonds (D^M) rises.

c. Premium due to tax exempt interest is $P^{M''} - P^{T''}$.

Term Structure of Interest Rates

A. Basic Concepts

1. Bonds with the same risk, liquidity, and tax status can have different interest rates because they have different times to maturity.
2. Yield curve is a plot of bond yields with different maturity dates but the same risk, liquidity, and tax considerations.
    a. The yield curve is usually upward sloping (long-term interest rates are higher than short-term interest rates).
    b. Sometimes the yield curve becomes inverted (short-term interest rates are higher than long-term rates).

3. Any theory of the term structure should explain these facts.
    a. Interest rates on bonds of different maturities tend to move together (fact 1).
    b. The yield curve usually slopes upward (downward) when short-term interest rates are low (high) (fact 2).
    c. Most times, the yield curve is upward sloping (fact 3).
4. There are three popular theories to explain the term structure of interest rates.
    a. Expectations theory (explains facts 1 and 2, but not 3)
    b. Segmented markets theory (explains fact 3, but not facts 1 and 2)
    c. Liquidity premium theory (explains facts 1, 2, and 3 by combining aspects of the expectations and segmented markets theories)

B. Expectations Theory

1. The expectations theory states that the interest rate on a long-term bond will equal the average of people's expectations of the short-term interest rates over the life of the long-term bond.
2. This theory assumes bond buyers do not prefer one maturity over another. That is, bond holders consider bonds of different maturities to be perfect substitutes.
3. Example: Consider a 2-year bond, where $R_{2,t}$ is the annual interest rate on that bond.
    a. $R_{1,t}$ is today's interest rate on a 1-year bond.
    b. $R_{1,t+1}$ is next year's expected interest rate on a 1-year bond.

c. The relationships between $R_{2,t}$ and both $R_{1,t}$ and $R_{1,t+1}$.

$$(1 + R_{2,t})\times(1 + R_{2,t}) = (1 + R_{1,t})\times(1 + R_{1,t+1})$$

$$1 + 2\times R_{2,t} + (R_{2,t})^2 = 1 + R_{1,t} + R_{1,t+1} + R_{1,t}\times R_{1,t+1}$$

$$2\times R_{2,t} + (R_{2,t})^2 = R_{1,t} + R_{1,t+1} + R_{1,t}\times R_{1,t+1} \qquad (1)$$

d. Since $(R_{2,t})^2$ and $R_{1,t}\times R_{1,t+1}$ are extremely small, we simplify (1) by setting $(R_{2,t})^2 = 0$ and $R_{1,t}\times R_{1,t+1} = 0$.

$$2\times R_{2,t} = R_{1,t} + R_{1,t+1}$$

$$R_{2,t} = (R_{1,t} + R_{1,t+1})/2$$

e. Suppose $R_{1,t} = 2.5\%$ and $R_{1,t+1} = 3.5\%$. What is the interest rate on a 2-year bond?

$$R_{2,t} = (2.5 + 3.5)/2$$

$$R_{2,t} = 3.0\%$$

4. According to the expectations theory, the interest rate on a n-year bond ($R_{n,t}$) is

$$R_{n,t} = \frac{R_{1,t} + R_{1,t+1} + R_{1,t+2} + \cdots + R_{1,t+n-1}}{n}$$

5. Expectations theory:

   a. Explains why interest rates on different maturities move together (fact 1).

   b. Explains why the yield curve is upward (downward) sloping when short-term interest rates are low (high) (fact 2).

   c. Cannot explain why the yield curve is upward sloping most of the time (fact 3).

C. Segmented Markets Theory

1. This theory assumes there are separate and segmented markets for each bond maturity.
2. Bond holders do not consider bonds of different maturities to be substitutes and instead have strong preferences for one maturity over another. People usually prefer shorter maturities because they have less interest rate risk.
3. The bond price and interest rate for each bond maturity are determined by the supply and demand for bonds at that maturity. Demand is usually higher for short-term maturities, which raises the prices and lowers the interest rates for those maturities.

4. Segmented markets theory:
    a. Explains why the yield curve is upward sloping most of the time (fact 3).
    b. Cannot explain why interest rates on different maturities move together (fact 1).
    c. Cannot explain why the yield curve is upward (downward) sloping when short-term interest rates are low (high) (fact 2).

D. Liquidity Premium Theory

1. The liquidity premium theory states that the interest rate on a n-year bond equals the average of expected short-term interest rates over those n years plus a positive liquidity premium ($l_{n,t}$).
2. People prefer to hold shorter-term bonds, so the size of the liquidity premium rises as the time to maturity increases.
3. According to the liquidity premium theory, the interest rate on a n-year bond ($R_{n,t}$) is

$$R_{n,t} = \frac{R_{1,t} + R_{1,t+1} + R_{1,t+2} + \cdots + R_{1,t+n-1}}{n} + l_{n,t}$$

4. Example: Find the interest rate on a 3-year bond, where $R_{1,t} = 2.5\%$, $R_{1,t+1} = 3.5\%$, $R_{1,t+2} = 4.5\%$, and $l_{3,t} = 0.5\%$.

$$R_{3,t} = \frac{2.5 + 3.5 + 4.5}{3} + 0.5 = 4.0\%$$

5. Liquidity premium theory:
    a. Explains why interest rates on different maturities move together (fact 1).
    b. Explains why the yield curve is upward (downward) sloping when short-term interest rates are low (high) (fact 2).
    c. Explains why the yield curve is upward sloping most of the time (fact 3).
6. The liquidity premium theory essentially combines expectations theory and the segmented markets theory.