Online Appendix to "The Political Economy of African Currency Unions: Evidence from a Time-Inconsistent Model"

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This online appendix presents the calibrated values for the covariance between the aggregate supply shock of the country i, $\varepsilon_{i,t}$, and the GDP-weighted average aggregate supply shock of the monetary union, $\varepsilon_{A,t}$, and derives the equations used to calibrate the utility parameters ϕ_g, ϕ_τ, and ϕ_π.

A Appendix

A.1 Covariances of the Aggregate Supply Shocks

Tables A.1 to A.3 display the covariance of an individual country's aggregate supply shock, $\varepsilon_{i,t}$, with the monetary union's aggregate supply shock, $\varepsilon_{A,t}$.

Table A.1: Cov($\varepsilon_{i,t}, \varepsilon_{A,t}$) for an Existing or Proposed Monetary Union

	WAEMU	CAEMC	WAMZ	WAEMU + WAMZ	CAEMC + WAMZ
Burkina Faso	0.0024	n.a.	n.a.	0.0017	n.a.
Cote D'Ivorire	0.0085	n.a.	n.a.	0.0063	n.a.
Guinea-Bissau	0.0033	n.a.	n.a.	0.0024	n.a.
Mali	0.0036	n.a.	n.a.	0.0038	n.a.
Niger	0.0002	n.a.	n.a.	0.0013	n.a.
Senegal	0.0027	n.a.	n.a.	0.0027	n.a.
Togo	0.0010	n.a.	n.a.	0.0005	n.a.
Cameroon	n.a.	0.0059	n.a.	n.a.	0.0026
Rep. of Congo	n.a.	0.0182	n.a.	n.a.	0.0043
Gabon	n.a.	0.0198	n.a.	n.a.	0.0110
The Gambia	n.a.	n.a.	−0.0004	−0.0009	−0.0004
Ghana	n.a.	n.a.	0.0019	0.0033	0.0021
Guinea	n.a.	n.a.	−0.0013	−0.0016	−0.0013
Liberia	n.a.	n.a.	0.0106	0.0086	0.0125
Nigeria	n.a.	n.a.	0.0032	0.0007	0.0025
Sierra Leone	n.a.	n.a.	0.0036	0.0001	0.0037

Table A.2: Cov($\varepsilon_{i,t}, \varepsilon_{A,t}$) from a Single WAMZ Country Joining WAEMU

	WAEMU +					
	The Gambia	Ghana	Guinea	Liberia	Nigeria	Sierra Leone
Burkina Faso	0.0024	0.0022	0.0024	0.0024	−0.0007	0.0023
Cote D'Ivorire	0.0083	0.0061	0.0077	0.0083	−0.0012	0.0076
Guinea-Bissau	0.0032	0.0022	0.0026	0.0030	−0.0003	0.0026
Mali	0.0036	0.0033	0.0033	0.0039	0.0005	0.0033
Niger	0.0002	0.0010	−0.0000	0.0005	0.0011	0.0005
Senegal	0.0027	0.0019	0.0024	0.0020	0.0004	0.0025
Togo	0.0010	0.0007	0.0008	0.0009	−0.0004	0.0008
The Gambia	−0.0004	n.a.	n.a.	n.a.	n.a.	n.a.
Ghana	n.a.	0.0028	n.a.	n.a.	n.a.	n.a.
Guinea	n.a.	n.a.	0.0008	n.a.	n.a.	n.a.
Liberia	n.a.	n.a.	n.a.	0.0166	n.a.	n.a.
Nigeria	n.a.	n.a.	n.a.	n.a.	0.0018	n.a.
Sierra Leone	n.a.	n.a.	n.a.	n.a.	n.a.	−0.0013

Table A.3: $\text{Cov}(\varepsilon_{i,t}, \varepsilon_{A,t})$ from a Single WAMZ Country Joining CAEMC

	CAEMC +					
	The Gambia	Ghana	Guinea	Liberia	Nigeria	Sierra Leone
Cameroon	0.0058	0.0035	0.0047	0.0061	0.0028	0.0058
Rep. of Congo	0.0178	0.0123	0.0165	0.0198	0.0040	0.0171
Gabon	0.0192	0.0145	0.0046	0.0207	0.0115	0.0194
The Gambia	−0.0004	n.a.	n.a.	n.a.	n.a.	n.a.
Ghana	n.a.	0.0042	n.a.	n.a.	n.a.	n.a.
Guinea	n.a.	n.a.	0.0099	n.a.	n.a.	n.a.
Liberia	n.a.	n.a.	n.a.	0.0352	n.a.	n.a.
Nigeria	n.a.	n.a.	n.a.	n.a.	0.0027	n.a.
Sierra Leone	n.a.	n.a.	n.a.	n.a.	n.a.	0.0059

A.2 Calibrating the Utility Parameters

Country i's optimal values for inflation, $\pi_{i,t}$, taxes, $\tau_{i,t}$, and government spending, $g_{i,t}$, under monetary independence are

$$\pi_{i,t} = \frac{\phi_g \phi_\tau \mu \overline{g}_i + (\phi_\tau + \phi_g + \mu\phi_g)c + \phi_\pi(\phi_\tau + \phi_g)(\pi_i^* - \delta\varepsilon_{i,t})}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}, \tag{A.1}$$

$$\tau_{i,t} = \frac{\phi_g \phi_\pi \overline{g}_i - (\phi_g\mu(1+\mu) + \phi_\pi)c - \mu\phi_g\phi_\pi(\pi_i^* - \delta\varepsilon_{i,t})}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}, \tag{A.2}$$

$$g_{i,t} = \frac{\phi_g(\phi_\pi + \mu^2\phi_\tau)\overline{g}_i + (\phi_\tau\mu - \phi_\pi)c + \mu\phi_\tau\phi_\pi(\pi_i^* - \delta\varepsilon_{i,t})}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}. \tag{A.3}$$

Country i's optimal values for inflation, $\pi_{MU,t}$, in a monetary union is

$$\pi_{MU,t} = \frac{\phi_g \phi_\tau \mu \overline{g}_A + ((\phi_\tau + \phi_g)(1-\theta_A) + \mu\phi_g)c + \phi_\pi(\phi_\tau + \phi_g)(\pi_{MU}^* - \delta\varepsilon_{A,t})}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}. \tag{A.4}$$

We begin by finding country i's expected values for inflation, $E_{t-1}[\pi_{i,t}] = \pi_i$, taxes, $E_{t-1}[\tau_{i,t}] = \tau_i$, and government spending, $E_{t-1}[g_{i,t}] = g_i$, under monetary independence:

$$\pi_i = \frac{\phi_g \phi_\tau \mu \overline{g}_i + (\phi_\tau + \phi_g + \mu\phi_g)c + \phi_\pi(\phi_\tau + \phi_g)\pi_i^*}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}, \tag{A.5}$$

$$\tau_i = \frac{\phi_g \phi_\pi \overline{g}_i - (\phi_g\mu(1+\mu) + \phi_\pi)c - \mu\phi_g\phi_\pi\pi_i^*}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}, \tag{A.6}$$

$$g_i = \frac{\phi_g(\phi_\pi + \mu^2\phi_\tau)\overline{g}_i + (\phi_\tau\mu - \phi_\pi)c + \mu\phi_\tau\phi_\pi\pi_i^*}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau + \phi_g)}. \tag{A.7}$$

Now, let us consider country j's values for π_j, τ_j, and g_j under monetary independence and subtract them from country i's values for π_i, τ_i, and g_i. The difference between those levels of optimal inflation, $\Delta\pi_{i,j} = \pi_i - \pi_j$, taxes, $\Delta\tau_{i,j} = \tau_i - \tau_j$, and government spending, $\Delta g_{i,j} = g_i - g_j$, are

$$\Delta\pi_{i,j} = \frac{\phi_g\phi_\tau\mu\Delta\overline{g}_{i,j} + \phi_\pi(\phi_\tau+\phi_g)\Delta\pi^*_{i,j}}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau+\phi_g)}, \tag{A.8}$$

$$\Delta\tau_{i,j} = \frac{\phi_g\phi_\pi\Delta\overline{g}_{i,j} - \mu\phi_g\phi_\pi\Delta\pi^*_{i,j}}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau+\phi_g)}, \tag{A.9}$$

$$\Delta g_{i,j} = \frac{\phi_g(\phi_\pi+\mu^2\phi_\tau)\Delta\overline{g}_{i,i} + \mu\phi_\tau\phi_\pi\Delta\pi^*_{i,j}}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau+\phi_g)}, \tag{A.10}$$

where $\Delta\overline{g}_{i,j} = \overline{g}_i - \overline{g}_j$ and $\Delta\pi^*_{i,j} = \pi^*_i - \pi^*_j$. Using (A.8), we solve for the ratio ϕ_π/ϕ_τ:

$$\frac{\phi_\pi}{\phi_\tau} = \frac{\left(\frac{\phi_g}{\phi_\tau}\right)\mu(\Delta\overline{g}_{i,j} - \mu\Delta\pi_{i,j})}{\left(1+\frac{\phi_g}{\phi_\tau}\right)(\Delta\pi_{i,j} - \Delta\pi^*_{i,j})}. \tag{A.11}$$

Similarly, (A.9) is solved for the ratio ϕ_π/ϕ_τ:[1]

$$\frac{\phi_\pi}{\phi_\tau} = \frac{\left(\frac{\phi_g}{\phi_\tau}\right)\mu^2\Delta\tau_{i,j}}{\left(\frac{\phi_g}{\phi_\tau}\right)(\Delta\overline{g}_{i,j} - \mu\Delta\pi^*_{i,j}) - \left(1+\frac{\phi_g}{\phi_\tau}\right)\Delta\tau_{i,j}}. \tag{A.12}$$

Since μ is assumed to be the same for all countries, the subtraction of the budget constraint for country j from the budget constraint for country i yields:

$$\Delta g_{i,j} = \mu\Delta\pi_{i,j} + \Delta\tau_{i,j}. \tag{A.13}$$

The values for ϕ_π/ϕ_τ in (A.11) and (A.12) are set equal to each other, and then the ratio ϕ_g/ϕ_τ is solved using (A.13) to simplify the answer:

$$\frac{\phi_g}{\phi_\tau} = \frac{\Delta\tau_{i,j}}{(\Delta\overline{g}_{i,j} - \Delta g_{i,j})}. \tag{A.14}$$

The values for μ and ϕ_g/ϕ_τ from (A.13) and (A.14), respectively, are substituted into (A.12):

$$\frac{\phi_\pi}{\phi_\tau} = \frac{(\Delta g_{i,j} - \Delta\tau_{i,j})\Delta\tau_{i,j}}{\Delta\pi_{i,j}(\Delta\pi_{i,j} - \Delta\pi^*_{i,j})}. \tag{A.15}$$

The expected value for inflation, $E_{t-1}[\pi_{MU,t}] = \pi_{MU}$, in a monetary union is

$$\pi_{MU} = \frac{\phi_g\phi_\tau\mu\overline{g}_A + ((\phi_\tau+\phi_g)(1-\theta_A) + \mu\phi_g)c + (\phi_\tau+\phi_g)\phi_\pi\pi^*_{MU}}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau+\phi_g)}. \tag{A.16}$$

Next, we calculate the difference between π_{MU} and π_i for country i:

$$\pi_{MU} - \pi_i = \frac{\phi_g\phi_\tau\mu(\overline{g}_A - \overline{g}_i) - \theta_A(\phi_\tau+\phi_g)c + (\phi_\tau+\phi_g)\phi_\pi(\pi^*_{MU} - \pi^*_i)}{\phi_\tau\phi_g\mu^2 + \phi_\pi(\phi_\tau+\phi_g)}, \tag{A.17}$$

[1] Alternatively, one could use (A.10) instead of (A.9) to solve for the ratio ϕ_π/ϕ_τ and still generate the same values for ϕ_g, ϕ_τ, and ϕ_π.

The parameter μ in (A.17) is defined as

$$\mu = \frac{(g_{MU} - g_i) - (\tau_{MU} - \tau_i)}{(\pi_{MU} - \pi_i)}, \tag{A.18}$$

where g_{MU} is the GDP weighted-average of government spending in the monetary union and τ_{MU} is the GDP weighted-average of taxes in the monetary union.[2] To put the utility parameters in ratio form, we divide (A.17) by ϕ_τ^2:

$$\pi_{MU} - \pi_i = \frac{\frac{\phi_g}{\phi_\tau}\mu(\overline{g}_A - \overline{g}_i) - \theta_A \frac{1}{\phi_\tau}(1 + \frac{\phi_g}{\phi_\tau})c + \frac{\phi_\pi}{\phi_\tau}(1 + \frac{\phi_g}{\phi_\tau})(\pi^*_{MU} - \pi^*_i)}{\frac{\phi_g}{\phi_\tau}\mu^2 + \frac{\phi_\pi}{\phi_\tau}(1 + \frac{\phi_g}{\phi_\tau})}. \tag{A.19}$$

One then solves (A.19) for ϕ_τ as a function of the ratios ϕ_π/ϕ_τ and ϕ_g/ϕ_τ:

$$\phi_\tau = \frac{\theta_A \left(1 + \frac{\phi_g}{\phi_\tau}\right) c}{\frac{\phi_g}{\phi_\tau}\mu\left((\overline{g}_A - \overline{g}_i) - \mu\left(\pi_{MU} - \pi_i\right)\right) - \frac{\phi_\pi}{\phi_\tau}(1 + \frac{\phi_g}{\phi_\tau})\left((\pi_{MU} - \pi_i) - (\pi^*_{MU} - \pi^*_i)\right)}, \tag{A.20}$$

where ϕ_g/ϕ_τ, ϕ_π/ϕ_τ, and μ are specified in (A.14), (A.15), and (A.18). Finally, we can substitute ϕ_τ into (A.14) and (A.15) to get ϕ_g and ϕ_π:

$$\phi_g = \frac{\Delta\tau_{i,j}}{(\Delta\overline{g}_{i,j} - \Delta g_{i,j})}\phi_\tau, \tag{A.21}$$

$$\phi_\pi = \frac{(\Delta g_{i,j} - \Delta\tau_{i,j})\Delta\tau_{i,j}}{\Delta\pi_{i,j}(\Delta\pi_{i,j} - \Delta\pi^*_{i,j})}\phi_\tau. \tag{A.22}$$

[2] The parameter μ in (A.13) is calculated as the difference between countries i and j, while the parameter μ in (A.17) is calculated as the difference between the weighted average of the monetary union and country i.